

Physics-Informed Neural Networks for the Time-Domain Maxwell Equations with Split-Field Perfectly Matched Layers

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Overview

1. Introduction
2. Maxwell equation with split-field PML
3. PINNs loss functions
4. Numerical results
5. Conclusion



1. Introduction

PINN Approach

- Mesh-Free: Eliminates tedious, complex 3D grid/mesh generation; works directly on a point cloud.
- Exact Derivatives: Uses Automatic Differentiation (AD) instead of error-prone numerical grid approximations.
- Naturally incorporates physical laws via Loss.
- Flexible for forward and inverse problems.

Especially, PML was not explored a lot combined with PINNs, but PML is used a lot in the industry.

2. Maxwell equation with split-field PML

$$\begin{aligned}\varepsilon_r \frac{\partial E_{zx}}{\partial t} + \sigma_x E_{zx} &= \frac{\partial H_y}{\partial x}, \\ \varepsilon_r \frac{\partial E_{zy}}{\partial t} + \sigma_y E_{zy} &= -\frac{\partial H_x}{\partial y}, \\ \mu_r \frac{\partial H_x}{\partial t} + \sigma_y^* H_x &= -\frac{\partial E_z}{\partial y}, \\ \mu_r \frac{\partial H_y}{\partial t} + \sigma_x^* H_y &= \frac{\partial E_z}{\partial x}.\end{aligned}$$

where ε_r , μ_r , σ and σ^* denote the relative electric permittivity, relative magnetic permeability, electric conductivity, and magnetic loss of the medium, respectively.

The above equations are nondimensional and are used in the PINN formulation to accelerate convergence

If $\sigma = 0$ and $\sigma^* = 0$, reduce to Maxwell's equations in a lossless medium.

$$\begin{aligned}\varepsilon_r \frac{\partial E_{zx}}{\partial t} + \sigma_x E_{zx} &= \frac{\partial H_y}{\partial x}, \\ \varepsilon_r \frac{\partial E_{zy}}{\partial t} + \sigma_y E_{zy} &= -\frac{\partial H_x}{\partial y}, \\ \mu_r \frac{\partial H_x}{\partial t} + \sigma_y^* H_x &= -\frac{\partial E_z}{\partial y}, \\ \mu_r \frac{\partial H_y}{\partial t} + \sigma_x^* H_y &= \frac{\partial E_z}{\partial x}.\end{aligned}$$

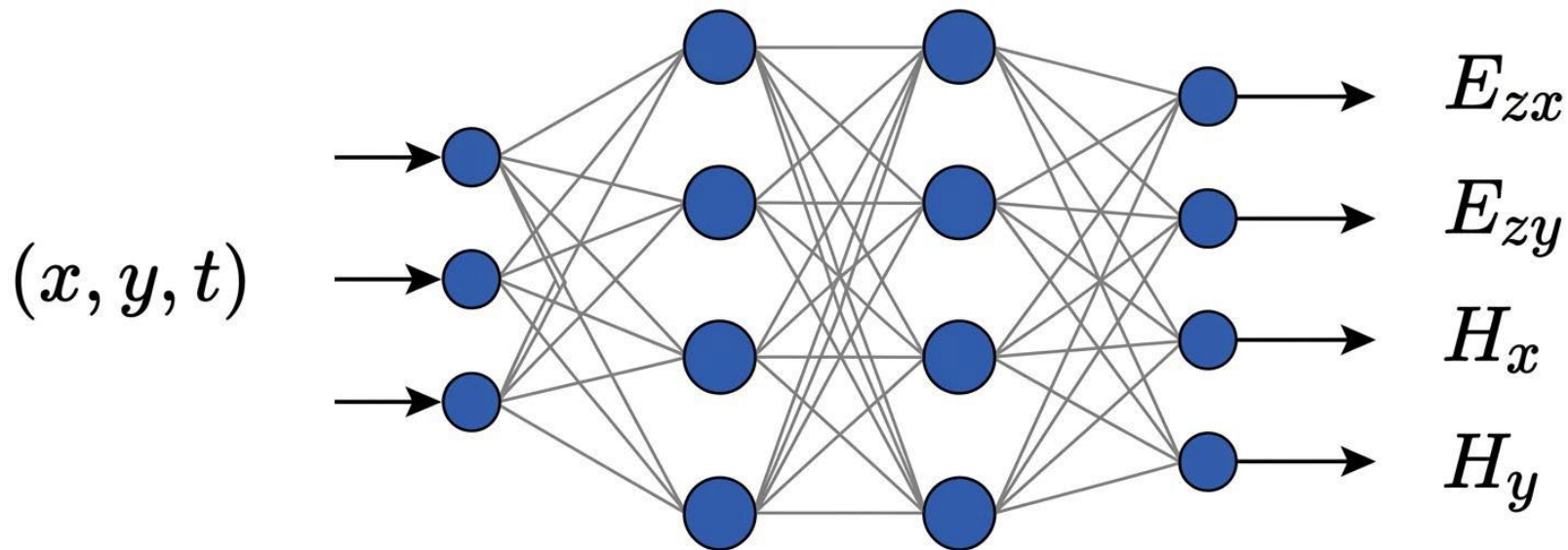
All computational domains, including PML region, can be represented by the same equation.

At the PML interface, the electric and magnetic loss parameter satisfy the standard PML impedance-matching condition, $\sigma/\varepsilon_r = \sigma^*/\mu_r$.

Taking σ_x as an example,

$$\sigma_x = \sigma_{\max} \left(\frac{\Delta x}{d} \right)^m$$

3. PINNs



Loss functions

The loss function L consists of several terms,

$$L = \lambda_1 L_{\text{physics}} + \lambda_2 L_{\text{initial}} + \lambda_3 L_{\text{boundary}} + \lambda_4 L_{\text{measure}}$$

$\mathcal{L}_{\text{physics}}$: Residual of PML equations

$\mathcal{L}_{\text{initial}}$: Error on initial field values

$\mathcal{L}_{\text{boundary}}$: Error on boundary field values
(especially at PML edges)

$\mathcal{L}_{\text{measure}}$: Error from other data

Here, the λ values denote the weights for each term



PINNs setup

- Framework: DeepXDE
- Sampling: Latin Hypercube Sampling (LHS)
- Activation: tanh (Hyperbolic tangent)
- Optimizer: 2-Stage Training, hybrid optimization strategy

Stage 1: Adam: First, for rugged global pretraining.

Stage 2: L-BFGS: Second, for fast, high-precision local convergence.



4. Numerical examples

1. 1D Gaussian Pulse

1. in vacuum
2. in multiple materials
3. with PML

2. 2D Gaussian Pulse in Vacuum with PML

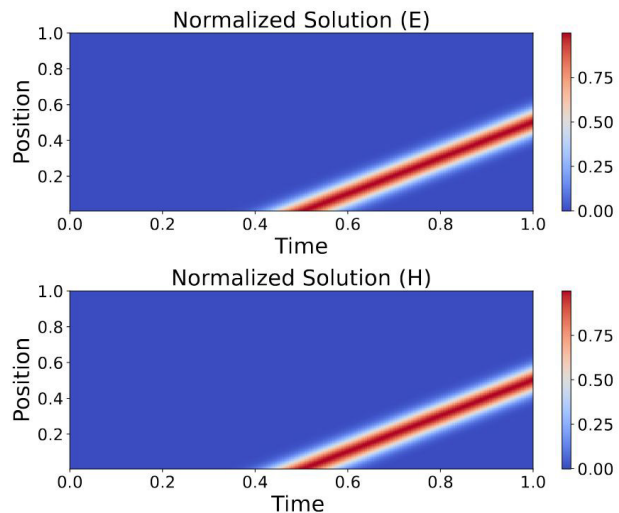
1. 1D Gaussian Pulse

The time-decaying Gaussian electric pulse is imposed as a Dirichlet boundary condition at the left boundary $z = 0$

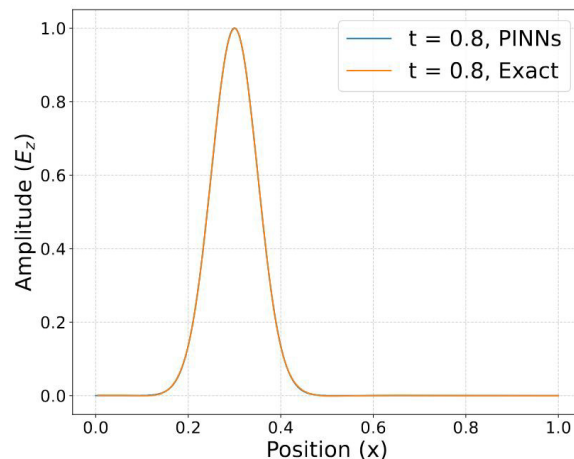
$$E_z(t) = e^{-\frac{(t-t_0)^2}{2\sigma^2}}$$

Here $t_0 = 0.5$, $\sigma = 0.05$

1.1 1D Gaussian pulse propagating in vacuum.



(a)



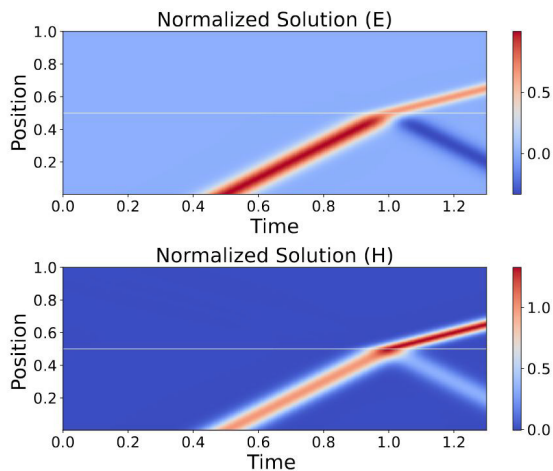
(b)

Figure 1: (a) Electric E_z and magnetic H_y fields. (b) Comparison between PINNs and the exact solution at $t = 0.8$.

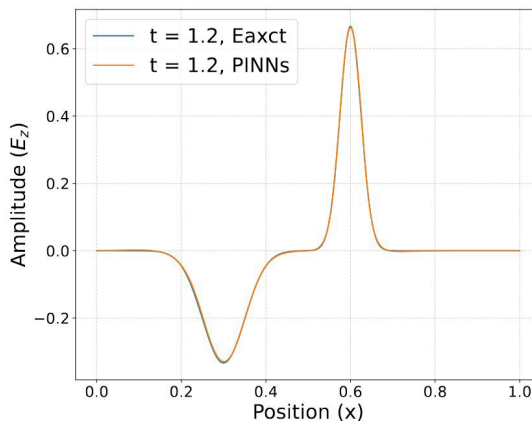
4 Layers,
40 Neurons
each layer

1.2 1D Gaussian pulse propagating in heterogeneous mediums.

A dielectric interface is located at $x = 0.5$, with the relative permittivity changing from $\epsilon_r = 1$ to $\epsilon_r = 4$.



(a)

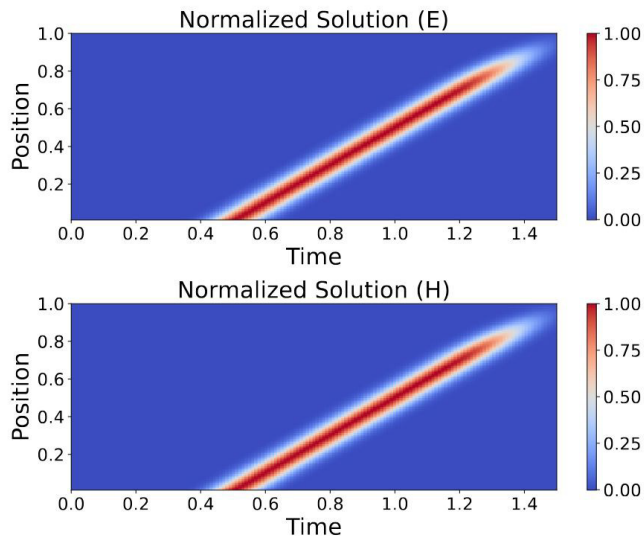


(b)

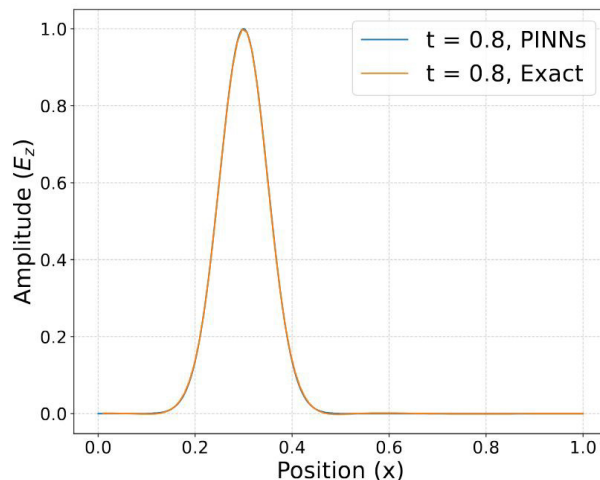
Figure 2: (a) Electric E_z and magnetic H_y fields. (The white line represent the interface at $x = 0.5$)
(b) Comparison between PINNs and the exact solution at $t = 1.2$.

4 Layers,
60 Neurons
each layer

1.3 1D Gaussian pulse wave propagating in vacuum with a PML region ($x > 0.6$)



(a)

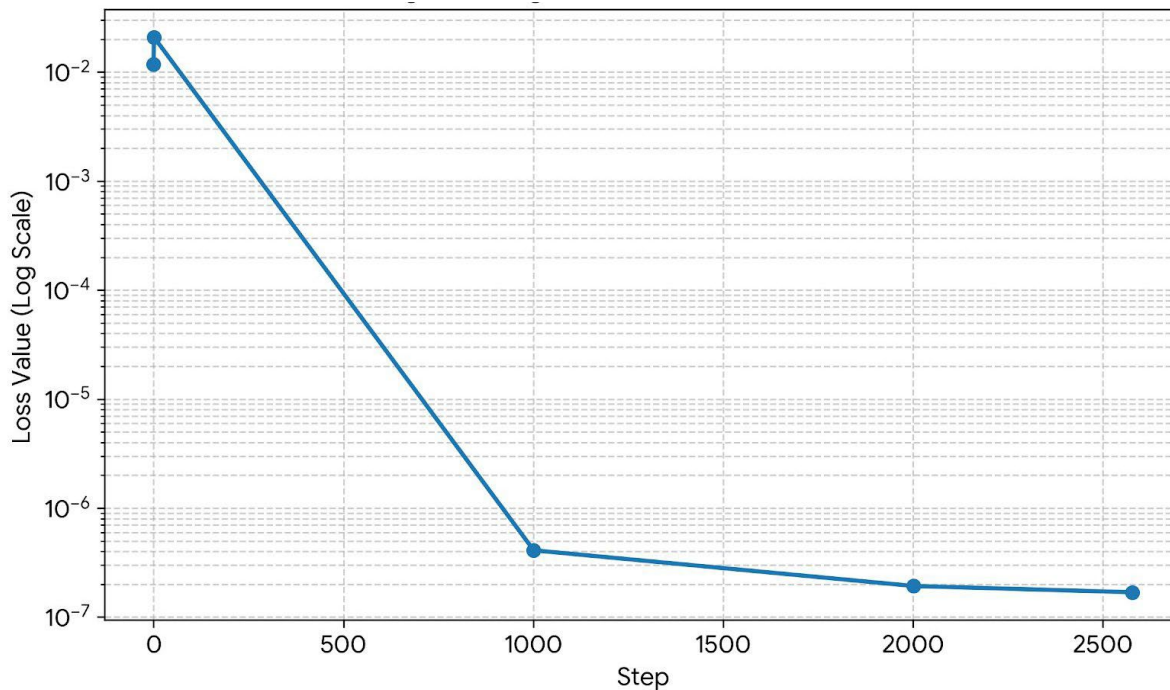


(b)

4 Layers,
60 Neurons
each layer

Figure 3: (a) Electric E_z and magnetic H_y fields. (b) Comparison between PINNs and FDTD at $t = 0.8$.

1.3 1D Gaussian pulse wave propagating in vacuum with a PML region ($x > 0.6$)



2. 2D Gaussian Pulse

This case considers the propagation of a 2D Gaussian pulse in vacuum. The computational space–time domain is $(x, y) \in [0.0, 1.0] \times [0.0, 1.0]$ with $t \in [0, 0.8]$. The PML interfaces are placed at $x = 0.2$, $x = 0.8$, $y = 0.2$, and $y = 0.8$

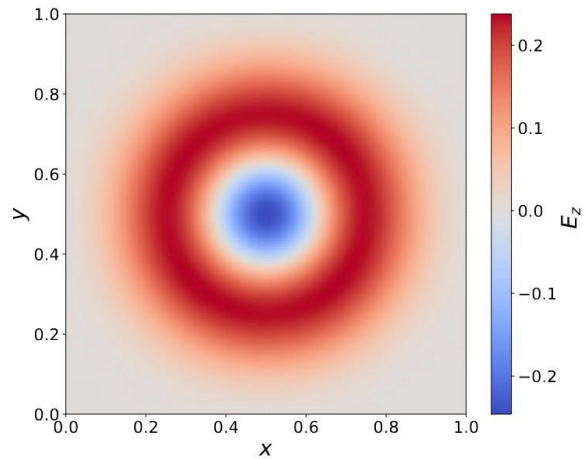
$$E_z(x, y, t = 0) = e^{-40((x-0.5)^2 + (y-0.5)^2)},$$

$$H_x(x, y, t = 0) = 0,$$

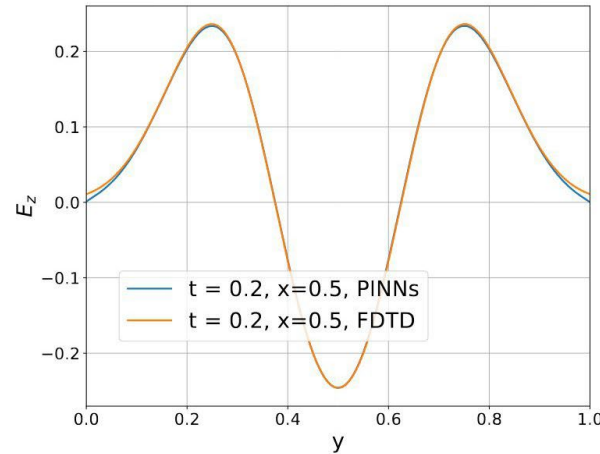
$$H_y(x, y, t = 0) = 0.$$

FDTD simulation was performed on a sufficiently large domain to obtain an accurate reference solution, without employing a PML.

4 Layers,
80 Neurons
each layer



(a)



(b)

Figure 4: (a) Electric field E_z at $t = 0.2$. (b) The solution comparison between PINNs and FDTD at $x = 0.5$ and $t = 0.2$.



The outward-propagating pulse is effectively absorbed by the PML region.

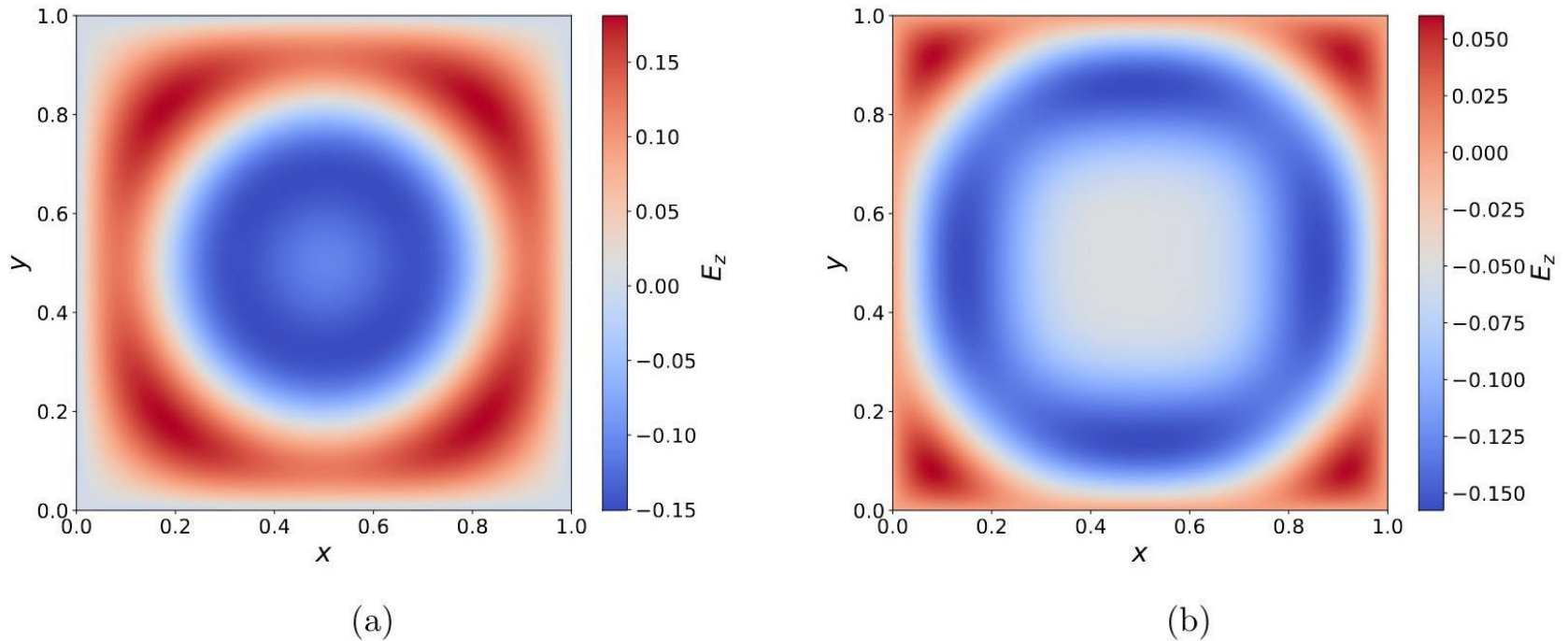


Figure 5: Electric field E_z at (a) $t = 0.4$; (b) $t = 0.56$.



5. Conclusions

This paper explored split-field PML in the context of PINNs. The results show good agreement with reference solutions, demonstrating the potential of PINNs combined with PML for practical electromagnetic problems.

Future work will focus on temporally discretized PINNs for solving higher-dimensional problems, multiple neural networks, etc.

More work needs to be done for current sheet excitation etc.



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